Correction of Gain Mismatch for Time Interleaved Analog to Digital Converter System

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***Abstract*— For digital signal processing, an analog signal should be sampled at least at the Nyquist rate. In many applications such as radar systems and software defined radios that deal with super high frequency, required sampling rates may exceed currently available analog-to-digital converters (ADCs). To increase the sampling rate time interleaved ADC (TI-ADC) systems are often used. In TI-ADC system several ADCs are interleaved to increase the sampling rate. Ideally all ADCs in the system will have the same gain. Due to imperfection of ADCs there is a gain mismatch. The gain mismatch results in spurious peaks in the spectrum and reduces the dynamic range of the TI-ADC system. In this paper we will first explain how the gain mismatch affect to the signal spectrum. We will also devise a method to estimate the gain mismatch and correct it.**

***Index Terms*— TI-ADC system, gain mismatch**

I. Introduction

For digital signal processing, a real continuous-time signal should be sampled at least at the Nyquist rate which is twice the highest frequency present in the signal. However, for super-high frequency signals such sampling rate may exceed that of currently available ADCs. To increase the sampling rate time interleaved ADC (TI-ADC) systems are often used. In TI-ADC system several ADCs are interleaved to increase the sampling rate. Ideally all ADCs in the system will have the same gain. Due to imperfection of ADCs there is a gain mismatch. The gain mismatch results in spurious peaks in the spectrum and reduces the dynamic range of the TI-ADC system [6]−[11]. In this paper we will first explain how the gain mismatch affect to the spectrum. We will also devise a method to estimate the gain mismatch and correct it. The easiest way to detect the gain mismatch is to use a known dc input. However, ADCs in the TI-ADC system often have different dc offsets. Different dc offsets of ADCs complicate the gain mismatch detection. One option for removing the dc offset is to AC-couple (using a highpass filter) to the input of the ADC [12]. AC-couple decouples dc so that there is no dc offset. Decoupling of dc results in losing some low frequency signal and increasing transient time but causes no effect to high frequency signals.

Accurate estimation of the gain mismatch is a key step in calibration of TI-ADC channel mismatches. We use a sinusoidal signal at Nyquist frequency to detect and correct the gain mismatch.

The paper is organized as follows. In section II, the Discrete Time Fourier Transform (DTFT) of a sampled sequence obtained by TI-ADC with the gain mismatch is derived and compared to the DTFT of a sequence without any gain mismatch. In section III, experimental result for detecting the gain mismatch using a sinusoidal signal at Nyquist frequency is presented. Finally, a conclusion is made in section VI.

II. DTFT of Sampled Signal with Gain Mismatch

TI-ADC system [1]−[5] is shown in Fig. 1.

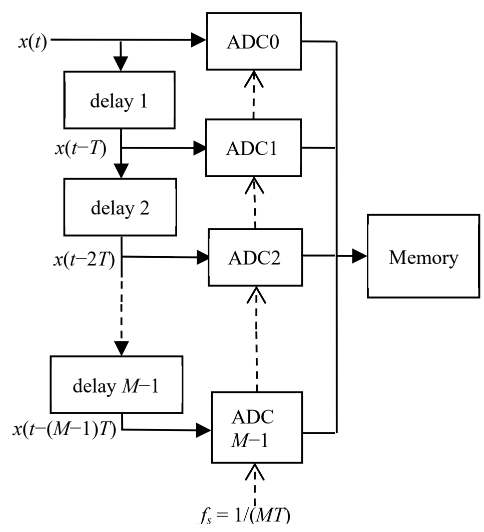

Fig. 1. TI-ADC system.

Multiple ADCs are used to increase the sampling rate. Suppose that there are M ADCs and the sampling rate of each ADC is $1/MT$ [Hz]. Then the resulting sampling rate of the TI-ADC system is $1/T$ [Hz]. Ideally gains of all ADCs are the same. However, the actual gain of the m-th ADC is given by G_m. This system results in gain mismatch.

For illustration let us choose $M = 3$ so that three ADCs are used in the TI-ADC system. After sampling with 3 ADSs in the TI-

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ADC system, samples with the gain mismatch and samples without any gain mismatch are illustrated in Fig. 2.

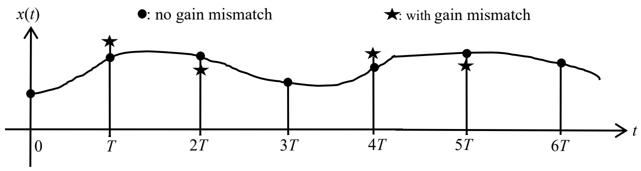

Fig. 2. G_m is the gain of m-th ADC. The number of ADCs, M, is 3. In this case, $G_0 = 1$. G_1 and G_2 are usually unknown.

A continuous-time signal $x(t)$ is sampled at times

$$(Mk + m)T \tag{1}$$

where k goes from 0 to $N/M - 1$ and m ranges from 0 to M−1, T is the sampling interval in second so that the number of samples is N which is assumed to be divisible by M.

Let us assume that the signal to be sampled is a complex exponential signal

$$x(t) = e^{j\theta_0 t}\,. \tag{2}$$

Assume that the signal is sampled uniformly with the sampling interval $T = 1$ [s], then the resulting sequence will be

$$x(n) = e^{j\theta_0 n} \tag{3}$$

for n = 0, 1, ⋯, N−1. The derivation of the DTFT of the sequence $x(n)$ is shown in APPENDIX I.

$$X(\theta) = \frac{\sin\left((\theta - \theta_0)N/2\right)}{\sin\left((\theta-\theta_0)/2\right)} e^{-j(\theta-\theta_0)(N-1)/2} \tag{4}$$

Now sampling of $x(t)$ with the gain mismatch results in the sequence

$$x_{mis}(Mk+m) = G_m e^{j\theta_0 (Mk+m)} \tag{5}$$

for $k = 0, 1, \cdots, \frac{N}{M} - 1$ and $m = 0, 1, \cdots, M-1$.

The derivation of the DTFT of the sampled sequence with the gain mismatch, $x_{mis}(n)$, is shown in APPENDIX II.

$$X_{mis}(\theta) = P(\theta)Q(\theta) \tag{6}$$

where

$$P(\theta) = \frac{\sin\left((\theta-\theta_0)N/2\right)}{\sin\left((\theta-\theta_0)M/2\right)} e^{-j(\theta-\theta_0)(N-M)/2} \tag{7}$$

and

$$Q(\theta) = \sum_{m=0}^{M-1} G_m e^{-j(\theta-\theta_0)m} \tag{8}$$

When $\theta_0 = 2\pi(0.2)$ [rad], the magnitudes of $X(\theta)$, $P(\theta)$, $Q(\theta)$, and $X_{mis}(\theta)$ for M = 3 are shown in Fig. 3. The gains were chosen as $G_0 = 1$, $G_1 = 1.2$, and $G_2 = 0.8$.

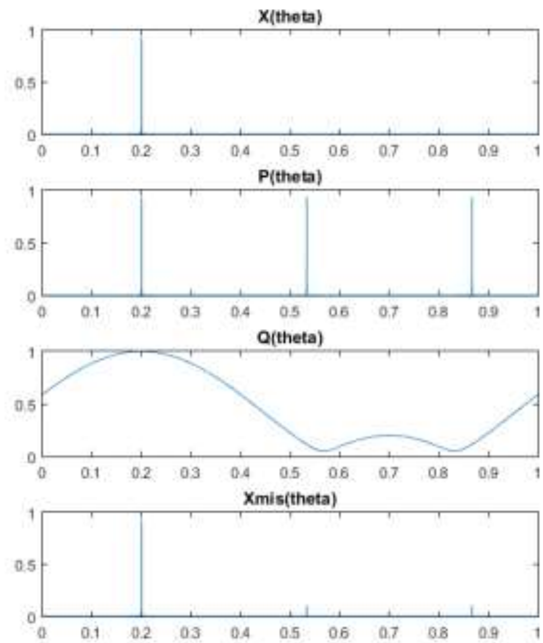

Fig. 3. The magnitudes of $X(\theta)$, $P(\theta)$, $Q(\theta)$, and $X_{mis}(\theta)$ when θ_0 = $2\pi(0.2)$ [rad]. Gains were $G_0 = 1$, $G_1 = 1.2$, and $G_2 = 0.8$. (Frequency in the horizontal axis is multiplied by 2π.)

The DTFT of the sequence without any gain mismatch $X(\theta)$ has the main lobe at $2\pi(0.2)$ [rad] only. The DTFT of the sequence with the gain mismatch, $X_{mis}(\theta)$, has two side lobes at $2\pi(0.2+\frac{1}{3})$ [rad] and $2\pi(0.2+\frac{2}{3})$ [rad] in addition to the main lobe at $2\pi(0.2)$ [rad]. There are two spurious peaks that distorts the spectrum.

We use a sinusoidal signal at Nyquist frequency to detect the gain mismatch. Nyquist frequency is half of the Nyquist rate. In this case the Nyquist frequency is

$$\frac{1}{2T}\ \text{[Hz]} \tag{9}$$

The sinusoidal is given by

$$x_{nyq}(t) = \cos(2\pi \times \frac{1}{2T} t + t_0) \tag{10}$$

If the test input signal is sampled at every T seconds, the resulting sequence will be

$$x_{nyq}(n) = \cos(\pi n + t_0) = (-1)^n \cos(t_0) \tag{11}$$

The absolute value of the resulting sequence is

$$|x_{nyq}(n)| = |\cos(t_0)| \tag{12}$$

for all n when there is no gain mismatch. When there is gain mismatch

$$|x_{nyq}(Mk + m)| = G_m|\cos(t_0)| \tag{13}$$

for $k = 0, 1, \cdots, \frac{N}{M} - 1$ and $m = 0, 1, \cdots, M-1$. By measuring the absolute value of the sampled signal we can detect the actual gain of each ADC and remove the gain mismatch by correcting the gains to be one for all ADCs.

III. EXPERIMENTAL RESULTS

For our experiment, the following FM signal is used.

$$x(t) = \cos\left(2\pi f_0 t + \sin(2\pi f_m t) + \phi\right) + e(t) \tag{14}$$

where $e(t)$ is the zero-mean Gaussian noise with the standard deviation of 0.01. We added a white noise to simulate the quantization noise of ADCs. The center frequency of the FM signal is chosen to be f_0 = 0.2 [GHz] and f_m = 10 [MHz]. The signal is sampled at the sampling rate of 1 [GHz] and 600 samples were taken. Because the signal is a real valued signal, the highest frequency of the signal that does not result in aliasing is 0.5 [GHz]. With M = 3, the gains were chosen as G_0 = 1, G_1 = 1.2, and G_2 = 0.8.

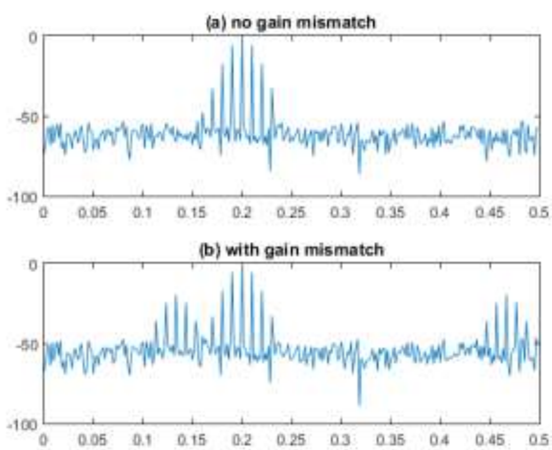

Fig. 4. Plot of the spectra in dB of the sampled FM signal (a) without gain mismatch and (b) with gain mismatch. The center frequency, f_0 = 0.2 [GHz].

The spectra in dB of the sampled FM signal without any gain mismatch and with the gain mismatch are shown in Fig. 4. In Fig. 4 (a), the peak main lobe is at the frequency of 0.2 [GHz]. In Fig. 4 (b) there are two peak sidelobes: one at (−0.2 + 1/3) [GHz] and the other at (−0.2 + 2/3) [GHz]. These spurious peaks are due to the gain mismatch and hamper the usage of TI-ADC systems.

To detect the gain mismatch, we input a sinusoidal signal with the Nyquist frequency which in this case is 0.5 [GHz]. The signal is sampled at the rate of 1 [GHz] with and without gain mismatch. 30 samples of the magnitude of the sequenc are plotted in Fig. 5.

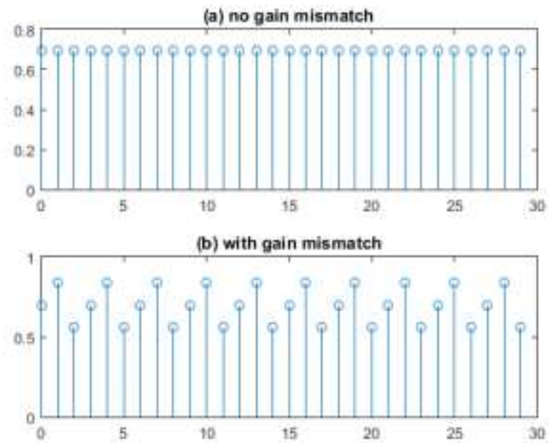

Fig. 5. Absolute value of the sampled Nyquist frequency signal using time interleaved ADC system (a) without gain mismatch, and (b) with gain mismatch.

When there is no gain mismatch, the absolute value of the sequence is constant at $|\cos(t_0)|$. When there is gain mismatch, the absolute values of ADC outputs are as follows as shown in Fig. 5 (b).

ADC0 output = $G_0|\cos(t_0)|$
ADC1 output = $G_1|\cos(t_0)|$
ADC2 output = $G_2|\cos(t_0)|$

From the absolute values of the output, the unknown gains G_0, G_1 and G_2 can be easily detected. Now the sampled FM signal can be reconstructed using the gains that are detected. The spectrum of the reconstructed signal is shown in Fig. 6.

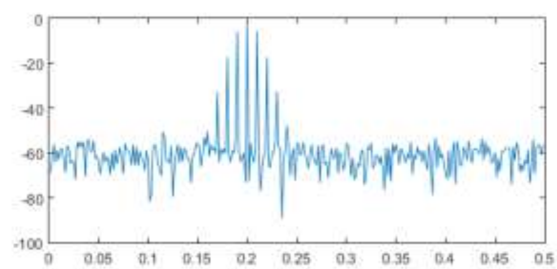

Fig. 6. Spectrum of the reconstructed FM signal using the detected gains

As one can observe from Fig. 6, spurious peaks were removed from the spectrum of the reconstructed signal. This method should work well for signals with sampling of super high frequency signals.

IV. Conclusion

In this paper we use a sinusoidal signal at the Nyquist frequency to detect the gain of each individual ADC in the TI-ADC system to correct the gain mismatch. The method works well for applications that do not deal with dc or low frequency signals.

Appendix I

The DTFT of the sampled sequence without gain mismatch $x(n)$ is

$$X(\theta) = \sum_{n=0}^{N-1} e^{j\theta_0 n} e^{-j\theta n} = \sum_{n=0}^{N-1} e^{-j(\theta-\theta_0)n}$$

$$= \frac{\left(e^{j(\theta-\theta_0)N/2} - e^{-j(\theta-\theta_0)N/2}\right) e^{-j(\theta-\theta_0)N/2}}{\left(e^{j(\theta-\theta_0)/2} - e^{-j(\theta-\theta_0)/2}\right) e^{-j(\theta-\theta_0)/2}}$$

$$= \frac{\sin\left((\theta-\theta_0)N/2\right)}{\sin\left((\theta-\theta_0)/2\right)} e^{-j(\theta-\theta_0)(N-1)/2}$$

Appendix II

If N is multiple of M, then the DTFT of the sampled sequence with mismatch gain mismatch $x_{mis}(n)$ of equation (5) is

$$X_{mis}(\theta) = \sum_{k=0}^{\frac{N}{M}-1} \sum_{m=0}^{M-1} G_m e^{j\theta_0(Mk+m)} e^{-j\theta(Mk+m)}$$

$$= \sum_{k=0}^{\frac{N}{M}-1} \sum_{m=0}^{M-1} G_m e^{-j(\theta-\theta_0)(Mk+m)} = P(\theta)Q(\theta)$$

where

$$P(\theta) = \frac{\sin\left((\theta-\theta_0)N/2\right)}{\sin\left((\theta-\theta_0)M/2\right)} e^{-j(\theta-\theta_0)(N-M)/2}$$

and

$$Q(\theta) = \sum_{m=0}^{M-1} G_m e^{-j(\theta-\theta_0)m} .$$

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