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Heterogeneous Behaviors and Direct Interactions in Artificial Stock Markets

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***Abstract*—In this paper, we explore the dynamic of stock prices over time by developing an agent-based market. The developed artificial market is comprised of heterogeneous agents occupied with various behaviors and trading strategies. The developed market has property of direct interaction. The environment takes the form of network structurenamely, it manifests as a scale-free network. The information will flow between the agents through the linkages that connect them. The model is subjected to testing for goodness of fit to the empirical observations of the S&P500. Furthermore, the effect of increasing the population size of various agent types is investigated.**

***Keywords*-agent-based model, stock market, heterogeneous agents, network topology, behavioral finance**

I. Introduction

The stock market is a complex system occupied by heterogeneous agents with different objectives, resources and abilities. Researchers have attempted to simulate the stock market in order to capture emergent phenomena of the market dynamics. The design of the simulated market involves the design of the agents (investors) attributes, the pricing mechanism, the market's environment, traded assets, and the methodologies for calibration and validation. Extensive literature surveys on the problem of the artificial stock market and related studies may be found in [1][2][3][4].

The limitation in humans' capabilities and the human psychology plays an important role in the infringement of the traditional assumption of the "rational man". Different sorts of feelings may influence our decisions and drive our behaviors to be irrational in the eyes of conventional economics. For example, Tversky and Kahneman [5] have confronted the risk aversion assumption by the advancement of prospect theory. Their theory experimentally showed that individuals have a tendency to emphasize losses more than profits and are therefore more loss-averse than risk-averse. Since the emergence of prospect theory, numerous behavioral biases have been addressed, reflecting that traders are made up from a very diverse set of types and behaviors. The most observed characteristics descriptive of the biased behavioral pattern in the trading space are indicated by [6][7][8].

Few studies have included behavioral biases in the context of the agent-based model e.g.,[9][10][11][12]. In our model we have focused on overconfidence, conservatism, and loss-averse biases

The agents' buying/selling decisions may be influenced by factors other than the agents' behaviors. In particular, they may be impacted by other agents' decisions. The interactions between agents are represented through social networks. Popular structures of networks includes random, distance-based, ring lattice, small world, and scale-free (power law) networks [13][14].

The objective of the research is to answer the following questions:

- What is the effect of increasing the population size of risk-averse and of loss-averse investors on market's volatility?
- What is the effect of increasing the population size of overconfident and of conservative investors on market's volatility?

II. Structure of the Market

The model consists of two entities that are represented in two hierarchical levels: the stock market (macro level) and the investors (micro level).

In the macro level, we represent the trading environment in term of network topologies. The trading environment can be structured as a random network, distance based network, small world network, and scale-free network. In addition, the macro level includes the market regulator. The market regulator in our model control the market prices through risk-free rates and tax imposed on transactions. If the risk-free rate or tax is incremented, the investors will incline to invest more in the risk-free asset and less in the risky asset. In addition, the market regulator may impose restrictions on the positions held by investors. In other words, they may inhibit the maximum quantity that the investors can short sell or acquire.

The main component of the micro levels is the agents (investors). The investors are characterized based on compensation for their preferences, behaviors, and trading strategies. The incorporated preferences and behaviors are risk-averse and loss-averse, with conservative and overconfident behaviors. The investment strategies are naive strategy (zero-intelligence agents), value (fundamental) investors, momen-

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tum traders, or investors that use artificial neural networks (ANNs). In the following, we described the macro and micro levels of market structure.

A. Macro Level

1) Network Structure: The environment can be represented in terms of network structure. In our model, we have utilized the scale-free network to symbolize the environment of the market. A major property of the scale-free network is that the degree distribution follows a power law [15]. The construction of the scale-free network follows preferential attachment algorithm developed by Barabási and Bonabeau [16][17]. The initial number of hubs in the network is controlled by a parameter, (H). As H increases, the number of hubs in the network increases.

2) Market Regulator: The regulatory forces of the stock market try to control the market by utilizing a variety of available tools such as imposing tax (c) on transactions and risk-free rates (r_f). The regulatory forces will monitor the market occasionally, where they evaluate the market returns according to the probability of execution (G). The market regulator attempts to prevent the market from abnormal bubbles or unexpected crashes; thus, if the return of the market increases above $r_f + \theta$, they will increment the tax on transactions by δc. Conversely, if the market return decreases below $r_f + \theta$, the tax on transactions will decrease by $\frac{1}{\delta}$. Increasing the transactions tax will drive the investors to invest more in bonds, and vice versa.

3) Fundamental Value of the Stock: The stock pays a stochastic dividend. The dividend stochastic process follows a Geometric Brownian motion with drift. However, the volatility in the process is inconsistent. The variance may be estimated using GARCH (1,1). We used discounted cash flow (DCF) models to estimate the underlying value of the stock at time t. Discounted cash flow models are based on the concept that the value of a share of stock is equal the present value of the cash flow that the stockholder expects to receive at time t [7] [18]. Using Williams model [19] the fundamental value of the stock at time t can be estimated as a function of dividend over the risk-free rate.

$$p_t^f = \frac{d_t}{r_f} \tag{1}$$

where p_t^f is the fundamental value of the stock and r_t is the risk free rate.

B. Micro Level Environment

1) Preferences and Behaviors: The overconfident and conservative behaviors are integrated in our model. Overconfident investors react aggressively to the change in the market; they firmly believe in their forecast and thus buy (sell) more shares. On the other hand, conservative investors react slowly to the market and change positions less frequently than overconfident or risk-averse investors do. In addition to that, loss-averse investors are included in the model. These are individuals who emphasize the losses more than gains. In other words, equal dollar amounts of loss or gain have unequal psychological impacts on the same agent. More impact is assigned to losses than to gains. The agents attempt to maximize their utility function subject to wealth constraints. The investor optimal holding can be represented in the following equation:

$$x_{i,t}^* = \frac{E_{i,t}\left(p_{t+1} + d_{t+1}\right) - \left(1 + r_f\right) p_t}{\lambda_i v_i \beta_i \sigma^2_{i,t,p_{t+1}+d_{t+1}}} \tag{2}$$

where $x_{i,t}$ is the number of shares that investor hold, t is the time index, and i is the index for the agents. λ is the risk aversion coefficient, v is the confidence/conservative coefficient and β is the loss aversion coefficient. p_t is the market price, and $E_{i,t}\left(p_{t+1} + d_{t+1}\right)$ is the expected price and dividend for the next time step. It is crucial for determination of the optimal holding. The expectations of the agents are heterogeneous and determined based on the investment strategies. $\sigma^2_{i,t,p_{t+1}+d_{t+1}}$ is the conditional variance of price and dividend at time $t + 1$.

2) Investment Strategies:

- Zero-intelligence:

Zero-intelligence traders are naive traders who do not follow a particular investment strategy. They randomly form their expectations of price movements. The zero-intelligence prediction is drawn from a uniform distribution around a reference price.

- Value (Fundamental) investors:

The value (fundamental) investors evaluate the stock in terms of its real value and pay less attention to the market's trend. They compare the intrinsic price with the current market price. If the fundamental price is higher than the current price they long the stock, otherwise they will short. Agents that use fundamental strategy know the process of the dividend but not the true fundamental value, and they form their expectation around the fundamental value of the stock.

- Momentum investors:

Momentum is a popular type of technical trading. Momentum traders follow the trend of the stock price. If the price is trending upward, they will think that it will continue to do so. Conversely, if the price movement downwards they will sell. The magnitude of the up or down movement of price and dividend is the basis of the agent future expectation. The expectation is heterogeneous among momentum traders. We assume that the momentum traders expect that the returns (of price and dividends) of the next period to be in the same direction of the current period.

- Artificial Neural Network Investors:

The agents that utilize ANNs update their beliefs according to the market states by optimizing the weights that connect the nodes in ANN layers.

Agents are assigned different initial weights, (w_{jk}), uniformly distributed between $[-1, 1]$. In addition, the structure of ANN varies among the agents. The number of hidden layer nodes varies from 1 to 10 nodes. The inputs to the network are a varied number of lagged returns r_{t-l} where $l = 0, ..., 10$. The output node is the expected return at the subsequent time step $\hat{r}_{t+1}$.

The historical returns that are used for learning process (L) varied among the agents, taking integer values between 10 and 100. This type of agent acclimates to the new market conditions by optimizing the weights in the neurons. Learning the optimal weights will permit them an opportunity to compete in the market. Nonetheless, the execution time for the learning procedure of neural network agents is not initiated at every time step, but rather controlled by a parameter, (K), that represents the probability of execution.

C. Investors' Interactions

Agents in this model have a direct interaction with each other. Once all agents settle on what they consider the optimal holding position, $x^*_{i,t}$, they share it with other agents in the agents connection group, which induces a direct connection with them. Based on the new information, each agent will update the final decision, where the final optimal holding position of stock shares, $X^*_{i,t}$. $X^*_{i,t}$, is a weighted average of his beliefs and the belief of the other agents in the connection group.

D. Adaptive Traits

The agents in our model would be able to sense information about the surrounding environment and other agents in their connection group. The adaptive traits in our market are limited to switching trading strategy and agent behavior. The decision-making process is not executed at every time step, but rather controlled by a parameter, (ω). This parameter controls how often the decision-making will be activated in a random manner. For example, if $\omega = 0.001$, then there is a low chance to activate the process of decision making for switching strategy.

Once the execution time is activated, the decision process will take place, and the agent will have to decide on the action. The decision will be focused around an examination of wealth. The agent will assess the wealth of other agents that have a direct connection with him. The agent will adapt to the trading strategy and the behavior of the agent with the most elevated wealth. Where the deciding agent is in fact the wealthiest, s/he will adhere to her own trading strategy and behavior.

E. Market Price Formation

The market price mechanism follows the price adjustment method [9] [20] [21][22]. Once each agent has settled on what he expects to be the optimal number of shares in his portfolio, $X^*_{i,t}$, he would send the number of shares that he wants to buy, $b^*_{i,t}$, or the number of shares that he wishes to sell, $o^*_{i,t}$, to the market.

$$b^*_{i,t} = \begin{cases} X^*_{i,t} - X_{i,t} & if\ X^*_{i,t} \geq X_{i,t} \\ 0 & Otherwise \end{cases} \tag{3}$$

$$o^*_{i,t} = \begin{cases} X_{i,t} - X^*_{i,t} & if\ X^*_{i,t} \leq X_{i,t} \\ 0 & Otherwise, \end{cases} \tag{4}$$

where $b^*_{i,t}$ is the required bid by agent i at time t , $X^*_{i,t}$ is the final required position by agent i at time t, $X_{i,t-1}$ is the actual position by agent i at time t, and $o^*_{i,t}$ is the required offer by agent i at time t.

The bids and offers of all agents will be summed at the market level:

$$B_t = \sum b_{i,t} \tag{5}$$

$$O_t = \sum o_{i,t}, \tag{6}$$

where B_t is the aggregate bids at time t, $b_{i,t}$ is the bid of agent i at time t, O_t is the aggregate offers at time t, and $o_{i,t}$ is the offer of agent i at time t.

The market price is determined using the price adjustment method:

$$p_t = p_{t-1}\left(1 + \eta\left(B_t - O_t\right)\right), \tag{7}$$

where p_t is the market price at time t, η is the speed adjustment of the price (sensitivity of the market), B_t is the aggregations of bids among all agents, and O_t is the total number of offers.

III. Calibration and Validation

A. Calibration

A model is a mapping of a real system into a representative system with parameters. There are two steps that enhance the approximation of the model to the real system. The first step is to calibrate the parameters to the output of the real system and finding the best-fit parameters of the model. Once the parameters are calibrated, the model with those parameters need to validated statistically to ensure that it is acceptable representation of the real system.

The agents-based models contain several parameters. These parameters may have uncertain values or a range of values, some of which may have a significant impact on the model behavior. In the calibration (or parametrization) step, we aim to optimize the model's parameters in order to engender a simulated time series of stock prices that is proximate to the observed empirical time series in the authentic market. It might be considered as the inverse problem of the simulation where instead of computing the outputs given the inputs and parameters, we determine the parameters given the inputs and outputs [23].

Table I
CALIBRATED POPULATION SIZE OF THE AGENTS

Population type	size	Population type	size	Population type	size
Risk averse-Z	7	Risk averse-OC-Z	10	Risk averse-C-Z	7
Risk averse-F	8	Risk averse-OC-F	7	Risk averse-C-F	8
Risk averse-M	10	Risk averse-OC-M	10	Risk averse-C-M	10
Risk averse-N	8	Risk averse-OC-N	10	Risk averse-C-N	9
Loss averse-Z	8	Loss averse-OC-Z	7	Loss averse-C-Z	11
Loss averse-F	9	Loss averse-OC-F	7	Loss averse-C-F	14
Loss averse-M	10	Loss averse-OC-M	7	Loss averse-C-M	7
Loss averse-N	9	Loss averse-OC-N	10	Loss averse-C-N	8

The adopted method for calibration is scatter search, an evolutionary method that has been successfully applied to hard optimization problems [24]. In order to calibrate the parameters, we need to identify the state vector of the market and map it to our simulation model. The state vector of the market is the frequency distribution of the returns of the S&P500 and the frequency distribution return from the 1000-day simulation. The objective is to minimize square error between the volatility and kurtosis of the S&P500 and the simulated market [25]:

$$min\ w_1 \left(\hat{\sigma} - \sigma\right)^2 + w_2 \left(\hat{K} - K\right)^2, \tag{8}$$

where $w_1 = 10,000$, $w_2 = 1$ are the weights assigned for each moment. The fitted distributions, returns, and price of the S&P500 and the simulated market are shown in Figure 1. However, we have calibrated the population of each type of investor. The distribution of investors populations' is shown in Table I. Likewise, the calibration of initial number of hubs, H, is 5, which demonstrates that 2.3% of the investors have the greater part of the associations.

Also, the adaptive parameters have been calibrated, where the calibration outputs show that the probability of switching trading strategy and investment behaviors is 0.003 and the update of the ANN weights occurred with probability $L = 0.01$.

Furthermore, the threshold for the regulator to interfere is high; they do not interfere unless the return rises or drops by more than 0.03. The regulator monitors the market at each time step, with probability of 10% (i.e., the likelihood of activating the taxation is $G = 0.078$). However, when the regulators interfere, they may increment the tax by 20%, where $\delta = 1.2$. The rest of the parameters are kept altered at $\lambda = 4$, $v_o = 0.7$, $v_c = 3$, $\beta = 2.5$, and $\alpha = 0.75$.

B. Validation

Calibrating the parameters does not mean that the model is ready to be deployed. Statistical validation should be enforced to ensure model validity. The statistical validation will be conducted against statistical properties of stock movements. These properties are fat tail of the returns, the auto-correlation of the returns ARCH effect [1]. Heteroskedasticity of stock volatility indicates that the variability of volatility is not equal among the values of prediction variable, which may create volatility cluster phenomena[26][27]. We have calculated the moments of the returns. Table II displays the moments of the artificial market returns and the S&P500. However, we have tested for excess kurtosis and asymmetry using Jarque–Bera test. The test outcomes demonstrate that the χ^2 values of the artificial market and the S&P500 are 1383.7 and 1323.7, respectively. The p-values were $2.2e - 16$ and $2.2e - 16$ for the artificial market and S&P500, respectively. These results permit us to reject the normality presumption for log-returns safely. The returns of both the S&P500 and the simulated market resolve fat tail and asymmetry.

[1]ARCH stands for Auto-regressive Conditional Heteroskedasticity

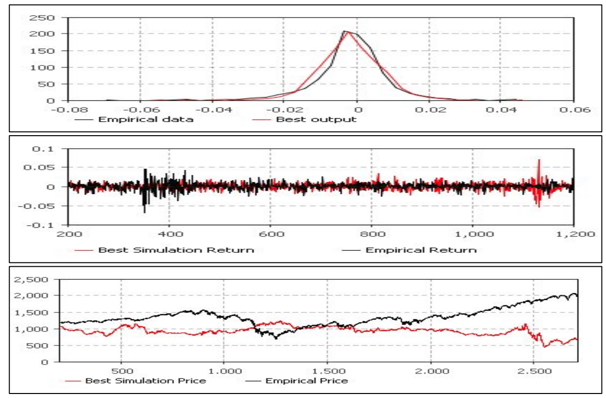

Figure 1. The fitted distributions, returns, and prices of the S&P500 and simulated market

The auto-correlation has been tested using Portmanteau statistics. Consequently, the χ^2 values for the simulation and S&P500 were 75.291 and 86.53, respectively, and the p-values were $9.2e - 6$ and $2.2e - 16$, respectively. These values fortify the rejection and the null hypothesis. The auto-correlation of returns function in figure 2 supports this conclusion.

Table II
THE MOMENTS OF THE SIMULATION AND THE S&P500

	μ	σ	S	K
S&P500	$4.92E-4$	0.00977	-0.577	5.685
ASM	$2.2E-4$	0.0091	0.084	5.619

Furthermore, the ARCH effect has been tested by using the Lagrange multiplier test. The χ^2 and p-values were 773.93 and 930.25, respectively, and $2.2e{-}16$ and $2.2e{-}16$, respectively, for the simulation and S&P500, respectively. These values suggest the rejection of the null hypothesis. Accordingly, we infer that stock returns of the artificial market and the S&P500 are exposed to ARCH effect phenomena. Additionally, the auto-correlation of returns square function in Figure 2 supports this conclusion.

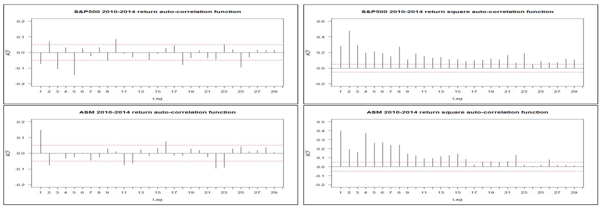

Figure 2. The auto-correlation functions of S&P500 and simulated market

Table III
SUMMARY OF THE EXPERIMENTS OUTPUTS

	Risk averse majority	Loss-averse majority	Overconfident majority	Conservative majority
Mean return	$9.2E-4$	$6.9E-4$	$2.3E-4$	$3.9E-4$
Volatility	0.00967	0.00959	0.0162	0.00616
Volatility Level	Medium	Medium	High	Low
Skewness	0.335	0.975	0.225	0.45
Kurtosis	4.2	4.97	6.13	5.24
Normality test	$\chi^2 = 756.4$ Not normal	$\chi^2 = 1197.5$ Not normal	$\chi^2 = 1581$ Not normal	$\chi^2 = 1182.5$ Not normal
Auto-correlation test	$\chi^2 = 63.4$ Auto-correlated	$\chi^2 = 126.49$ Auto-correlated	$\chi^2 = 283.5$ Auto-correlated	$\chi^2 = 639.69$ Auto-correlated
ARCH effect test	$\chi^2 = 1301.1$ ARCH effect exists	$\chi^2 = 825.01$ ARCH effect exists	$\chi^2 = 1807.9$ ARCH effect exists	$\chi^2 = 1133.8$ ARCH effect exists

IV. FACTOR EFFECTS ON MARKET DYNAMICS

By validating the model, we have provided an experimental environment that allows us to investigate the impact of various factors on the dynamics of the market. Recall that in this research we aim to investigate the effect of increasing the population size of certain type of agents. The first experiment recognizes the market patterns when the majority of agents (around 70% of the population) are risk averse. The majority of the population in the second experiment is loss-averse. In the third and fourth experiments, the majorities of the populations are overconfident and conservative investors, respectively. The prices and returns pattern of the experiments are shown in Figure 3.

The most elevated mean volatility is observed when the market filled-out with overconfident investors, where the annualized volatility was 25.6%. The volatility of a market with a majority of overconfident investors is higher than that of the S&P500 and the simulated market for the calibrated period.

On the other hand, the average volatility diminished as the proportion of conservative investors increased. The average annualized volatility of a market dominated by conservative investors was 9.7%. This quality is near the S&P500 under a low volatility regime. Additionally, for the market with risk- and loss-averse investors, the average volatiles were 15.2% and 15.1%, respectively. These outputs demonstrate no significant difference between the S&P500 and the calibrated market.

V. CONCLUSION AND FURTHER RESEARCH

In this paper, we have developed an agent-based model with heterogeneous trading behaviors. The model incorporates overconfident, conservative, and loss-averse behaviors alongside the traditional risk-averse investors. The investors in the model may utilize sundry investment strategies such as fundamental, technical, advanced adaptive strategy such as an ANN, or simply speculate on the future prices in an arbitrary manner.

The investors interact directly with their living surroundings. The environment is represented by a scale-free network. Likewise, we have integrated the role of the regulator in the artificial market. The regulator appoints tax on the investors' transactions. The taxes increment/ decrement as the stock returns increase/ decrease.

The objective of this research was to calibrate the models parameters with the specific end goal of validating the major stylized facts of the empirical observations of the daily returns of the S&P500. We optimized agent population sizes for the goodness of fit of the returns volatility and kurtosis of the S&P500 for the period from Dec 2010 to Dec 2014. The parametrization was performed using a scatter search algorithm.

The validity of the model was checked against the empirical stylized facts of the S&P500 for the period from Dec 2010 to Dec 2014. The stylized facts of both the S&P500 and the artificial stock market show asymmetry in the return distribution. Additionally, the artificial market and the S&P500 exhibit some return and returns square autocorrelation at seven lag.

Moreover, four experiments were executed to explore the impact of expanding the populace size of the risk-averse, loss-averse, overconfident, and conservative investors. The experiment results show that the volatility of the market achieves the most when the investors are overconfident and drops down to the lowest level when the conservative investors represent the majority in the market. In the other two cases, the volatility levels settled at a medium level. In addition, the market was exposed to fat-tail phenomena in every experiment, which indicates nonappearance of market efficiency.

In future work, different sorts of investors may be investigated. Behaviors and biases such as optimistic, pessimistic, anchoring bias, mental accounting bias, and ambiguity-averse bias may be incorporated. In addition, the effect of different network structures on the dynamics of market prices may be investigated. Experimental designs may be implemented to study the impact of the behavioral coefficients on the market dynamic. Finally, we may examine the distribution of wealth among the agents under various scenarios.

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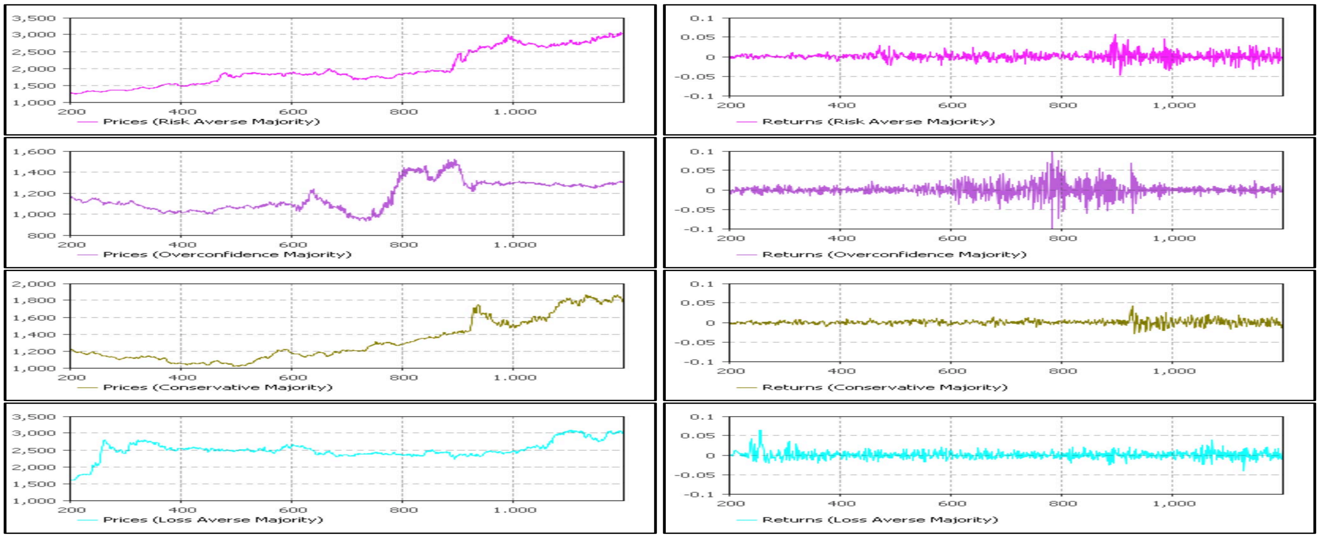

Figure 3. The market patterns for the performed experiments

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